Southwest Minnesota State University
Department of Mathematics

SMSU Mathematics Course Content

2012- 2013

The following is a list of possible topics and techniques to cover in your SMSU College Now course. This is NOT a comprehensive list of suggested content. We do not expect anyone teaching these classes to cover every topic on this list. Rather, to gather what content is currently being taught in the College Now courses, we would like to aggregate the following information. Please place a checkmark next to the content that you currently cover.

1 Math 110 College Algebra

1.1 Description:

Mathematics topics for students whose backgrounds are insufficient for them to begin their study of mathematics at a higher level. Topics include: equations and inequalities, functions, graphs, polynomials, systems of equations, matrices, and determinants. The required preparation is MATH 060 or two years of high school mathematics, including at least one year of algebra.

1.2 Possible Topics:

Integer Exponents
Rational Exponents
Real Exponents
Radicals
Polynomials
Factoring Polynomials
Rational Expressions
Complex Number
Solving Equations and Inequalities
Solutions and Solution Sets
Linear Equations
Applications of Linear Equations
Equations With More Than One Variable
Quadratic Equations
Applications of Quadratic Equations
Equations Reducible to Quadratic Form
Equations with Radicals
Linear Inequalities
Polynomial Inequalities
Rational Inequalities
Absolute Value Equations
Absolute Value Inequalities
Graphing
Lines
Circles
The definition of a Function
Graphing Functions
Combining functions
Inverse Functions
Piecewise Function
Parabolas
Ellipses
Hyperbolas
Transformations
Symmetry
Rational Functions
Polynomial Function
Dividing Polynomials
Roots of Polynomials
Graphing Polynomials
Finding Zeroes of Polynomials
Partial Fractions
Exponential Functions
Logarithm Functions
Solving Exponential Equations
Solving Logarithm Equations
Systems of Equations
Linear Systems with Two Variables
Linear Systems with Three Variables
Sequences
Series
Mathematical Induction
Augmented Matrices
Nonlinear Systems

1.3 Possible Techniques:

1.3.1. Fundamental concepts of Algebra

- ☐ Use scientific notation
- ☐ Use properties of exponents to solve algebraic equations
- ☐ Solve algebraic expressions
- ☐ Solve rational expressions

1.3.2. Equations and Graphs

- ☐ Sketch the graph of an equation
- ☐ Find x- and y-intercepts of a graph of an equation
- ☐ Use symmetry to sketch graph of an equation
- ☐ Find equations of and sketch graphs of circles
- ☐ Use graphs of equations in solving real-life problems
- ☐ Write and use mathematical models to solve real-

life problems

- ☐ Solve quadratic equations by:
  1. factoring
  2. extracting square roots,
  3. by completing the square,
  4. using the Quadratic Formula.

1.3.3. Complex Numbers

- ☐ Add, subtract, and multiply complex numbers
- ☐ Use complex conjugates to write the quotient of two complex numbers in standard form
- ☐ Find complex solutions of quadratic equations

1.3.4. Inequalities

- ☐ Represent solutions of linear inequalities in one variable
- ☐ Use properties of inequalities to create equivalent inequalities
- ☐ Solve linear inequalities in one variable
- ☐ Solve linear inequalities involving absolute values
- ☐ Use inequalities to model and solve real-life problems
- ☐ Solve polynomial inequalities
- ☐ Solve rational inequalities

1.3.5. Functions

- ☐ Find the slope of a line given two points on the line
- ☐ Use slope to identify parallel and perpendicular lines
- ☐ Use slope and linear equations in two variables to model and solve real-live problems
- ☐ Use function notation and evaluate functions
- ☐ Find the domains of functions
- ☐ Use functions to model and solve real-life problems
- ☐ Evaluate difference quotients
- ☐ Find the zeros of functions graphically
- ☐ Determine intervals on which function are increasing or decreasing
- ☐ Determine relative maximum and relative minimum values of functions
- ☐ Identify even and odd functions
- ☐ Identify and graph linear, squaring, cubic, square root, reciprocal and piecewise-defined functions
- ☐ Use vertical and horizontal shifts to sketch graphs of functions
- ☐ Use reflection to sketch graphs of functions
- ☐ Use nonrigid transformations to sketch graphs of functions
- ☐ Add, subtract, multiply, and divide functions
- ☐ Find the composition of functions
- ☐ Use combinations and compositions of function to model and solve real-life problems
- ☐ Find inverse functions and verify that two functions are inverse functions of each other
- ☐ Determine if a function is one-to-one

1.3.6. Polynomials

- ☐ Write quadratic functions in standard form and use the results to sketch graphs of functions
- ☐ Find minimum and maximum values of quadratic functions in real-life applications
- ☐ Use transformations to sketch graphs of polynomial functions
- ☐ Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions
- ☐ Find and use zeros of polynomial functions as sketching aids
- ☐ Use the Intermediate Value Theorem to help locate zeros of polynomial functions
- ☐ Use long division to divide polynomials by other polynomials
- ☐ Use synthetic division to divide polynomials by binomials of the form $(x - k)$
- ☐ Use the Fundamental Theorem of Algebra to determine the number of zeros of polygonal functions
- ☐ Find rational zeros of polynomial functions
- ☐ Find conjugate pairs of complex zeros
- ☐ Find zeros of polynomial by factoring
- ☐ Use Descartes's Rule of Sign to find zeros of polynomials
- ☐ Use the Rational Root Test to determine the rational roots of polynomials
- ☐ Use mathematical models to approximate sets of data points

1.3.7. Rational Functions

- ☐ Find the vertical and horizontal asymptotes of graphs of rational functions
- ☐ Use rational functions to model and solve real-life problems
- ☐ Analyze and sketch graphs of rational functions
- ☐ Recognize, graph, and write equations of the four basic conics: circle, ellipse, parabola, and hyperbola
- ☐ Recognize equations of conics that have been shifted vertically or horizontally in the plane and write and graph equations of conics that have been shifted vertically or horizontally in the plane

1.3.8. Exponential and Logarithmic Functions

- ☐ Recognize, graph, and evaluate exponential and logarithmic functions
- ☐ Use exponential and logarithmic functions to model and solve real-life problems
- ☐ Use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions

1.3.9. Systems of Equations and Inequalities

- ☐ Use the method of substitution and the method of elimination to solve systems of linear and nonlinear equations in two variables
- ☐ Use a graphical approach to solve systems of equations in two variables
- ☐ Use systems of equations to model and solve real-life problems
- ☐ Interpret graphically the numbers of solutions of systems of linear equations in two variables
- ☐ Use systems of linear equations in two variables

to model and solve real-life problems

- ☐ Use back-substitution to solve linear systems in row-echelon form
- ☐ Use Gaussian Elimination to solve systems of linear equations
- ☐ Solve systems of linear equations in three or more variables to model and solve real-life problems
- ☐ Find partial fraction decomposition of rational expression
- ☐ Sketch the graphs of inequalities in two variables
- ☐ Solve systems of inequalities
- ☐ Use systems of inequalities in two variable to model and solve real-life problems
- ☐ Solve linear programming problems
- ☐ Use linear programming to model and solve real-life problems

1.3.10. Matrices

- ☐ Perform elementary row operations on matrices
- ☐ Use matrices and Gaussian elimination to solve systems of linear equations
- ☐ Use matrices and Gauss-Jordan elimination to solve systems of linear equations
- ☐ Add and subtract matrices and multiply matrices by scalars
- ☐ Multiply two matrices
- ☐ Use matrix operations to model and solve real-live problems
- ☐ Use Gauss-Jordan elimination to find the inverse of a matrix
- ☐ Use inverse matrix to solve systems of linear equations
- ☐ Use Cramer's Rule to solve systems of linear equations
- ☐ Use determinates to find the areas of triangles
- ☐ Use a determinate to test for collinear points and find and equation of a line passing through two points
- ☐ Use matrices to encode an decode messages

1.3.11. Sequences and Series

- ☐ Use sequence notation to write the terms of sequences
- ☐ Use factorial notation
- ☐ Use summation notation to write sums
- ☐ Find the sum of a series
- ☐ Use sequences and series to model and solve real-life problems
- ☐ Find nth partial sums of arithmetic sequences
- ☐ Recognize, write, and find the nth terms of geometric sequences
- ☐ Find the sum of a finite or infinite geometric sequence
- ☐ Use mathematical induction to prove statements involving a positive integer n
- ☐ Recognize patterns and write the nth term of a sequence
- ☐ Use the Binomial Theorem to calculate binomial coefficients
- ☐ Use Pascal's Triangle to calculate binomial coefficients
- ☐ Use binomial coefficients to write binomial expansions

1.3.12. Counting and Probability

- ☐ Solve simple counting problems
- ☐ Use permutations and combinations to solve counting problems
- ☐ Find the probabilities of events
- ☐ Find the probabilities of mutually exclusive events
- ☐ Find the probabilities of independent
- ☐ Find the probability of the complement of a event

1.4 Key Theorems:

☐ **The Remainder Theorem**
If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

- ☐ Proof

☐ **The Factor Theorem**
A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

- ☐ Proof

☐ **Linear Factorization Theorem**
If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2)(x - c_3)\dots(x - c_n)$$

where $c_1, c_2, \dots, c_n$ are complex numbers.

- ☐ Proof

☐ **Factors of a Polynomial**
Every polynomial of degree $n > 0$ with real coefficients can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.

- ☐ Proof

☐ **Rational Root Test**
If the polynomial

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

has integer coefficients and if c/d is a rational zero of $f(x)$ such that c and d have no common prime factor, then

1. the numerator c of the zero is a factor of the constant term a_0
2. the denominator d of the zero is a factor of the leading coefficient a_n.

- ☐ Proof

☐ **Standard Equation of a Parabola**
The standard form of the equation of a parabola wither vertex at $(0,0)$ and directrix $y=-p$ is

$$x^2 = 4py, \quad p \neq 0$$

For directrix $x=-p$, the equation is

$$y^2 = 4px, \quad p \neq 0$$

The focus is on the axis p units from the vertex.

☐ Proof

☐ **Properties of exponential functions**
Let a be a positive number, n be a real number, and x and y be positive real numbers.

$$a^x a^y = a^{x+y} \quad (a^x)^y = a^{xy} \quad a^0 = 1 \quad a^{-n} = \frac{1}{a^n}$$

☐ Proof

☐ **Properties of logarithmic functions**
Let a be a positive number ($a \neq 1$), n be a real number, and u and v be positive real numbers.

$$\log_b(uv) = \log_b(u) + \log_b(v) \quad \log_b\left(\frac{u}{v}\right) = \log_b(u) - \log_b(v) \quad \log_b(u)^n = n\log_b(u)$$

☐ Proof

☐ **Properties of Sums**
Let c be a real number and n be a positive integer.

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i \qquad \sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

☐ Proof

1.5 Other